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Analysis of Particulate Removal in Venturi Scrubbers—Effect of Operating Variables on Performance

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A theoretical model of venturi scrubber performance consisting of the governing differential equations for momentum, material, and energy exchange is solved using a Milne fourth-order predictor—Hamming corrector numerical integration procedure. The model provides for specification of the operating variables, including scrubber geometry, throat gas velocity, liquid to gas loading ratio, and collector droplet and particle-size distributions. Liquid loading ratio and gas velocity are shown to be the two most important operating variables, while the dispersity of the droplet size distribution only slightly affects collection efficiency over the operating range normally encountered. Location of the liquid injection site and length of the throat are also important design considerations. While particle collection in venturi scrubbers has typically been assumed to occur in the potential flow regime, the present results show that collection can also occur under conditions corresponding to viscous flow about the collecting droplets. The transition Reynolds number used to change from the potential to viscous flow equations affects the predicted performance.

SCOPE

High energy scrubbers have long been useful for particulate removal and control, especially in the metallurgical and chemical process industries. Despite this generally successful use of the venturi, reliable design criteria are generally not available; there is considerable reliance on prior experience and pilot scale tests. The primary objective of the present study was to develop a realistic computer model that predicts particulate collection efficiency in a venturi scrubber.

The model requires specification of liquid flow rate, gas flow rate, venturi geometry, entering conditions of the liquid and gas, and particle size distribution. The dust particles are assumed to move along the gas streamlines and are removed by inertial impaction, interception, and other secondary mechanisms. The equations of motion of the collector droplets and the mass balances for the droplet and dust phases are numerically solved. The flux of the dust at the exit of the venturi is integrated over the cross-sectional area and compared to the dust input to determine the collection efficiency. The model is evaluated in computer simulations by varying the important parameters and comparing the results obtained with some of the limited data available in the literature.

CONCLUSIONS AND SIGNIFICANCE

In this research, a simulation model was developed to more realistically model venturi scrubber performance. In particular, allowance was made for a complete description of all operating variables, scrubber geometry, and parameters relating to properties of the droplet and dust size distributions. Following are specific conclusions pertaining to the results of this investigation.

1. The data with which to validate models describing venturi scrubber performance are extremely limited. The present model was compared to the data presented by Ekman and Johnstone (1951) and Brink and Contant (1958). In most cases, experimentally determined collection efficiencies were 10-35% of theoretical. It has been pointed out that the data of Ekman and Johnstone may not be typical of industrial scale venturi scrubbers due to the small size of their laboratory scale venturi (cf., Boll, 1973). Also, the data of Brink and Contant may have been taken under conditions where poor spray coverage in the throat section existed. In any event, the predic-

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tions of the current model are as good or better than those made by other models.

2. The model compares favorably with current analytical models such as that of Yung et al. (1978) when similar geometries and operating conditions are observed. It must be noted that these analytical models are very restrictive in terms of flexibility and are of little value in modeling effects such as heat and mass transfer or initial drop size distribution.

3. Incorporating the drop distribution believed to exist at the injection site may be more important for certain geometries or operating conditions. For example, in modeling the Brink and Contant scrubber, differences of 10-50% in performance were predicted by assuming a single drop size vs. multiple drop classes. For simulations of the Ekman and Johnstone scrubber, the observed differences were much smaller. The parametric studies showed that the effect of the major variables on collection efficiency was the same when comparing simulations involving a single drop size or multiple drop classes, although there were differences in the actual collection efficiencies predicted.

4. The liquid-loading ratio and the throat gas velocity most significantly affect venturi scrubber performance. This is due not only to their appearance in the relationship used to describe the characteristic droplet diameter assumed at the injection site, but also to their entering the equation describing the removal rate of particulate material through terms involving the relative velocity, the droplet concentration, and the target efficiency.

5. Other parameters also affect performance to various degrees:

a. The parameter δ in the Nukiyama-Tanasawa drop-size distribution function does not significantly affect scrubber performance in the range of $\delta = 1/4$ to $\delta = 1/2$. Distributions produced by other means with δ values outside of this range might significantly affect the efficiency of removing the particulate material.

b. The location of the injection site and the length of the throat section were both seen to be important geometry parameters. Models which cannot account for these two variables are of limited usefulness.

6. The collection efficiency can be correlated using the parameter $L\sqrt{\mathrm{St}}$. The parameter $N_{VH}\sqrt{\mathrm{St}}$ can also be used, although this parameter is less useful since the value of N_{VH}, the number of velocity heads, cannot generally be predicted in advance.

7. Particle collection can occur in the viscous regime under typical scrubber operation. Further work needs to be done to establish at what Reynolds number the transition from potential to viscous flow should be made, or to develop other means to account for this change in flow patterns around collectors.

In a subsequent paper, the significant effects of heat and mass transfer and particle collection by the interception mechanism will be discussed.

INTRODUCTION

This paper presents a model which realistically describes the removal of particulate material from gas streams by venturi scrubbers. The overall collection efficiency of a venturi scrubber is the result of a number of collection mechanisms operating simultaneously. These include inertial impaction and interception, and to a lesser extent, diffusion, electrostatic, and mass and heat transfer effects. The contribution of any one mechanism depends on the particle and droplet sizes and their relative velocity. These, in turn, are related to the scrubber geometry and operating conditions. In typical installations, a range of droplet and particle sizes are encountered, and the analysis is more complex due to the interactions between the separate classes of droplets and particles.

The venturi scrubber was first patented in 1925 but the modern version was not put into operation on an industrial scale until 1947. Since that time, a great volume of literature has been published dealing with the different aspects of venturi scrubber operation. Though in some cases equations were presented which provided guidelines to estimate performance, most of this work was based on assumptions which severely limited their general usefulness and accuracy.

The purpose of this study was to develop a realistic model for venturi scrubber performance. Particular attention has been paid to allow a complete description of all important operating variables and geometry, droplet and particle size distributions, the specification of inlet gas temperature and humidity, spray liquid temperature, and an accurate description of the physical properties. This research has also attempted to give insight to the basic nature of the collection process as a number of mechanisms operating simultaneously, and to evaluate the effects of mass and heat transfer. In a subsequent paper the role of mass transfer-induced diffusiophoretic collection in hindering or enhancing collection is specifically evaluated.

THEORY

Figure 1 shows the geometry used for the parametric studies described later. It has been adapted from the research of Boll (1973) dealing with the relationship between particle collection efficiency and pressure drop. Although smaller than a commercial scale venturi, it does represent an order-of-magnitude scale-up from most laboratory scale venturi scrubbers and should be free from many of the scale-up problems discussed by Taheri et al. (1973).

In most respects, venturi scrubbers can be reasonably well modeled using a one-dimensional analysis. In the cases of mass and heat transfer, no large gradients exist in the direction normal to flow due to the eddy activity in the main stream. Tillman (1971) has carried out a theoretical analysis on the effect of main-stream turbulence and found little increase in the probability of collision between droplets and dust particles due to turbulent fluctuations. Under some operating conditions, water distribution near the injection site may be far from uniform. Under these conditions, a more realistic model for the spray pattern will be required.

Liquid Atomization

As schematically shown in Figure 1, dust-laden gas from the process stream enters the venturi and is accelerated in the converging section. At the injection site, the high relative velocity between the gas and scrubbing liquid causes a violent disruption of the liquid into a distribution of spherical droplets. The particulate material suspended in the gas phase collides

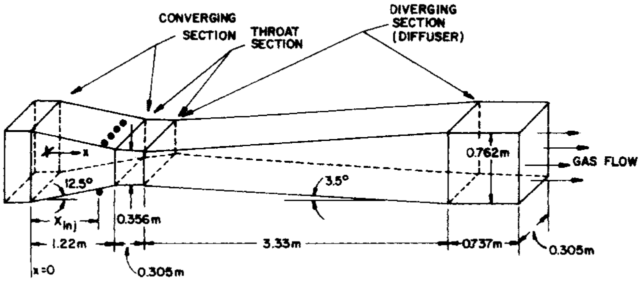

Figure 1. Schematic of venturi scrubber used for the present studies.

with the large, slower moving droplets through the various short-range collection mechanisms. Due to the drag forces, the droplets accelerate down the length of the venturi. Since the relative velocity decreases, the local collection efficiencies become smaller as the droplets move downstream. The gas decelerates in the diverging section, which provides for additional removal of the particulate material by collision with the now faster moving droplets.

The determination of the droplet size distribution from such pneumatic atomization processes is most difficult. An empirical equation often used in venturi scrubbers is that due to Nukiyama and Tanasawa (1938)

$$f(D) = \frac{\delta b^{3/\delta}}{\Gamma(3/\delta)} D^2 \exp(-bD^\delta) \tag{1}$$

where the parameters b and δ affect the dispersion and central value of the distribution. Based on experiments conducted on alcohol and glycerine being atomized by compressed air, Nukiyama and Tanasawa proposed an empirical equation to relate the mean droplet size to the physical properties and atomization conditions. When physical properties for the air-water system are employed, the equation becomes

$$\overline{D_{32}} = \frac{4.892 \times 10^{-3}}{V_{\mathrm{rel}}} + 0.01206\, L^{1.5} \tag{2}$$

where the Sauter mean diameter, $\overline{D_{32}}$, reflects the size of droplet which possesses the same volume to surface area ratio as the sum of all drops in the distribution. The Sauter mean diameter has been found to be useful in describing the efficiency of spray breakup processes and in describing mass transfer and particle collection processes (Mugele and Evans, 1951). It has been used as the single characteristic diameter in virtually all previous scrubber models. For a discrete distribution, the Sauter mean diameter can be calculated from the expression

$$\overline{D_{32}} = \frac{\sum_{j=1}^{j^*} D_j^3 f'(D_j)}{\sum_{j=1}^{j^*} D_j^2 f'(D_j)} \tag{3}$$

where $f'(D_j)$ is the fraction of drops in the interval having a characteristic diameter D_j.

For the Nukiyama-Tanasawa distribution function, the Sauter mean diameter is

$$\overline{D_{32}} = b^{-(1/\delta)} \frac{\Gamma(6/\delta)}{\Gamma(5/\delta)} \tag{4}$$

Recognizing the importance of modeling the nature of the droplet distribution, this study approximated the continuous nature of the distribution by a number of droplet diameter classes selected to retain the essential features of the actual distribution. Experiments by Lewis et al. (1948) on venturi atomizers have shown that in most cases $\delta = 1/4$ or $1/3$ provides a good representation of the distribution. For the majority of simulations described in this study, the value $\delta = 1/4$ was assumed.

Equations 2 and 4 can be used to provide a complete description of the droplet size distribution. This distribution is discretized to provide a number of droplet classes each with a known fraction of the total injected spray. Although the validity of the Nukiyama-Tanasawa distribution in describing the conditions in an operating venturi has been questioned by several investigations (Lewis et al., 1948; Boll et al., 1974), a universally accepted distribution function has not been advanced.

Particle-Size Distribution

The production of micron-sized particles by industrial processes can be characterized by a distribution function describing particle diameter. Generally, the particular process by which the particles are formed determines the size distribution. Since the present study assumes that there are no particle-to-particle interactions, the results are not dependent on any particular distribution, and for simplicity, a uniform distribution was assumed.

Equation of Motion for Single Droplet

For those cases where the only important external force is the drag force, the equation of motion for a single spherical droplet can be expressed as

$$\frac{d}{dt}(M_j\vec{u}_j) = \vec{F}_{B_j} + C_{D_j}\frac{\pi D_j^2}{8}\rho(\vec{v} - \vec{u}_j)|\vec{v} - \vec{u}_j| \tag{5}$$

For a one-dimensional analysis where body forces may be ignored, this becomes

$$M_j\frac{du_j}{dt} + u_j\frac{dM_j}{dt} = C_{D_j}\frac{\pi D_j^2}{8}\rho(v - u_j)|v - u_j| \tag{6}$$

This equation can be recast to change time derivatives into gradients in the axial direction yielding

$$M_ju_j\frac{du_j}{dx} + u_j^2\frac{dM_j}{dx} = C_{D_j}\frac{\pi D_j^2}{8}\rho(v - u_j)|v - u_j| \tag{7}$$

The second term will be zero for cases where mass transfer is unimportant.

If the droplet mass is expressed in terms of diameter, the equation of motion for a single droplet belonging to any class becomes

$$\frac{du_j}{dx} = \frac{3}{4}C_{D_j}\frac{\rho(v - u_j)|v - u_j|}{u_j\rho_s D_j} - \frac{3u_j}{D_j}\frac{dD_j}{dx}; j = 1, \ldots, j^* \tag{8}$$

The last term represents the momentum contribution from mass transfer.

The drag coefficient, C_{D_j}, is a function of the droplet Reynolds number. Boll (1973) presents a review of available correlations and data with emphasis on applicability to venturi scrubber operations. At the venturi throat and injection site, highly turbulent conditions exist. The effect of turbulence intensity on drag coefficient has been discussed by Clift and Gauvin (1971). However, this effect was not included in the present study. The drag coefficient was determined by considering the process to be at steady-state, and empirical relationships for the standard drag coefficient curve were used.

Equations for Droplet and Particle Concentration

For a system of noninteracting droplets under steady operating conditions, the equation of droplet continuity for a particular droplet class is

$$u_jN_jA - \left[u_jN_jA + \frac{\partial(u_jN_jA)}{\partial x}dx\right] = 0$$

or

$$\frac{\partial(u_jN_jA)}{\partial x} = 0 \tag{9}$$

Thus, the term u_jN_jA (representing the drop formation rate, $\dot{N}_j$) is a constant. The number concentration of droplets in any class is then

$$N_j = \dot{N}_j/u_jA; \qquad j = 1, \ldots, j^* \tag{10}$$

where $\dot{N}_j$ depends on the conditions which exist at the spray injection site.

In a similar fashion, one can develop an expression for particle continuity for the same control volume. The removal rate depends on the relative velocity between the particles and droplets, the local target efficiency, and the number concentration of particles and droplets. For steady operation, the governing equation is

$$vn_iA - \left[vn_iA + \frac{\partial(vn_iA)}{\partial x}\,dx\right]$$

$$- \sum_{j=1}^{j^*} (v - u_j)n_iA\,\frac{\pi D_j^2}{4}\,E_{ij}N_j dx = 0; \quad i = 1, \ldots, i^*$$

or

$$\frac{\partial(vn_iA)}{\partial x} = - \sum_{j=1}^{j^*} (v - u_j)n_iA\,\frac{\pi D_j^2}{4}\,E_{ij}N_j \tag{11}$$

The summation is required to account for collection of particles by each of the droplet classes.

The local target efficiency, E_{ij}, reflects the ease of collection of particles belonging to the i^{th} class by droplets belonging to the j^{th} class. It depends on the physical properties of the gas and liquid phases, the sizes associated with the droplet and particle classes, as well as various dynamic characteristics at the local position in the scrubber. Expressions have been developed to account for the various flow regimes including the effect of mass and heat transfer (Placek and Peters, 1980).

The primary variable expressing the performance of a venturi scrubber is the reduction of particle loading. Noting the gas flowrate is $Q_g = vA$, the input rate of dust belonging to the i^{th} class is

$$\dot{n}_i = n_iQ_g = n_ivA \tag{12}$$

From Eq. 11, the expression for the change in particle loading with respect to position is therefore

$$\frac{d\dot{n}_i}{dx} = - \sum_{j=1}^{j^*} (v - u_j)n_iAN_j\left(\frac{\pi}{4}\,D_j^2\right)E_{ij} \tag{13}$$

Variation of Gas Velocity in the Venturi

At standard atmospheric pressure, the humid volume of a parcel of moist air is given as (Treybal, 1968)

$$V_H = (0.00283 + 0.004551\,H)T_g \tag{14}$$

Assuming ideal gas behavior between the venturi inlet and an arbitrary position, the local gas velocity will be

$$v = \frac{Q_{g_o}}{V_{H_o}}\,\frac{(0.00283 + 0.004551H)T_g}{\alpha AP_T} \tag{15}$$

where the void fraction α is given by

$$\alpha = \frac{\dot{m}_s}{A\sum_{j=1}^{j^*} \rho_s f'_o(D_{jo})D_{jo}^3}\;\frac{\sum_{j=1}^{j^*} \rho_s f'(D_j)D_j^3}{\sum_{j=1}^{j^*} \rho_s f'(D_j)u_j} \tag{16}$$

The fraction of drops in the j^{th} class at the venturi inlet and at any arbitrary position are $f_o'(D_{jo})$ and $f'(D_j)$, where provisions must be made to change the characteristic diameter of an interval when mass transfer is occurring.

Mechanical Energy Balance

The pressure drop across a venturi scrubber is one of the primary performance parameters, and results from momentum exchange between the gas phase, the droplet phase, and the wall of the venturi. In deriving the mechanical energy equation, the presence of the dust phase is neglected. A momentum balance around a control volume with differential length, dx, has the form

$$A\,\frac{dP_T}{dx} + \dot{m}_g\,\frac{dv}{dx} + \sum_{j=1}^{j^*} \dot{m}_j\,\frac{du_j}{dx} + \frac{\dot{m}_s + \dot{m}_g}{\dot{m}_g}\,\frac{f\rho v^2A}{2D_h} = 0 \tag{17}$$

The first term represents pressure forces acting over the cross-sectional area, A; the second term represents the change in gas momentum; the third term is the change in droplet momentum for all droplet classes; and the last term accounts for momentum loss to the wall as shear.

In observing operating venturis, Boll (1973) noted that another momentum loss exists due to the formation of a liquid film on the venturi wall. It was pointed out that this is partly offset by the fact that the mixture density correction usually underestimates the wall friction term. Although Yoshida et al. (1960) have suggested the use of different f values in each venturi section, Boll (1973) found satisfactory results using the single value 0.027.

Combining Eq. 17 with the droplet equation of motion, the pressure gradient can be written as

$$\frac{dP_T}{dx} = -\rho v\,\frac{dv}{dx} - \sum_{j=1}^{j^*} \frac{3}{4}\,\frac{\rho}{\rho_s}\,\frac{\dot{m}_j}{D_jA}\,C_{Dj}\,\frac{(v - u_j)|v - u_j|}{u_j} + 3\,\frac{\dot{m}_ju_j}{D_jA}\,\frac{dD_j}{dx} - \frac{\dot{m}_s + \dot{m}_g}{\dot{m}_g}\,\frac{f\rho v^2}{2D_h} \tag{18}$$

Note that the first and second terms on the right hand side allow for either pressure loss or recovery. The third term is zero for operation in the absence of mass and heat transfer. Hollands and Goel (1975) have non-dimensionalized a form of the mechanical energy balance and presented nomographs to estimate the pressure drop. However, numerical integration of Eq. 18 was necessary in this study to obtain the axial pressure distribution.

Mass and Heat Transfer Effects

In an operating venturi mass and heat transfer effects will affect the validity of the equations developed by causing variations in the droplet diameters, gas temperature, velocity, humidity, and ultimately, the local target efficiency. These effects are fully discussed in a subsequent paper. This limits the current model to situations involving a saturated gas phase where droplet and gas temperatures are approximately the same.

RESULTS AND DISCUSSION

In this section, the results obtained from the theoretical model will be presented. Initially, the predictions are compared with the limited data having well-defined scrubber geometry and detailed particle information suitable for these comparisons. In addition, the results will be compared with previous theoretical models, in particular those of Boll (1973) and Yung et al. (1978). Finally, the effect of various operating parameters on performance will be discussed. The Milne fourth-order predictor-Hamming corrector numerical procedure was used to integrate the system of differential equations in all simulations.

Comparison with Other Studies

The performance of a scrubber in collecting particles belonging to the i^{th} size class may be represented by the overall particle collection efficiency, $E_{i_{ov}}$, which is defined in terms of the inlet and exit dust loading rates.

$$E_{i_{ov}} = 1 - (\dot{n}_i)_{\mathrm{exit}}/(\dot{n}_i)_{\mathrm{inlet}} \tag{19}$$

Equation 12 defines $\dot{n}_i$. Alternately, the performance can be expressed in terms of the penetration, $P_{i_{ov}}$,

$$P_{i_{ov}} = 1 - E_{i_{ov}} \tag{20}$$

or more conveniently, in terms of the number of transfer units, N_{Ti}, given by

$$N_{Ti} = \ln\left(\frac{1}{P_{Ti}}\right) \tag{21}$$

Each 2.303 transfer units is equivalent to an order of magnitude

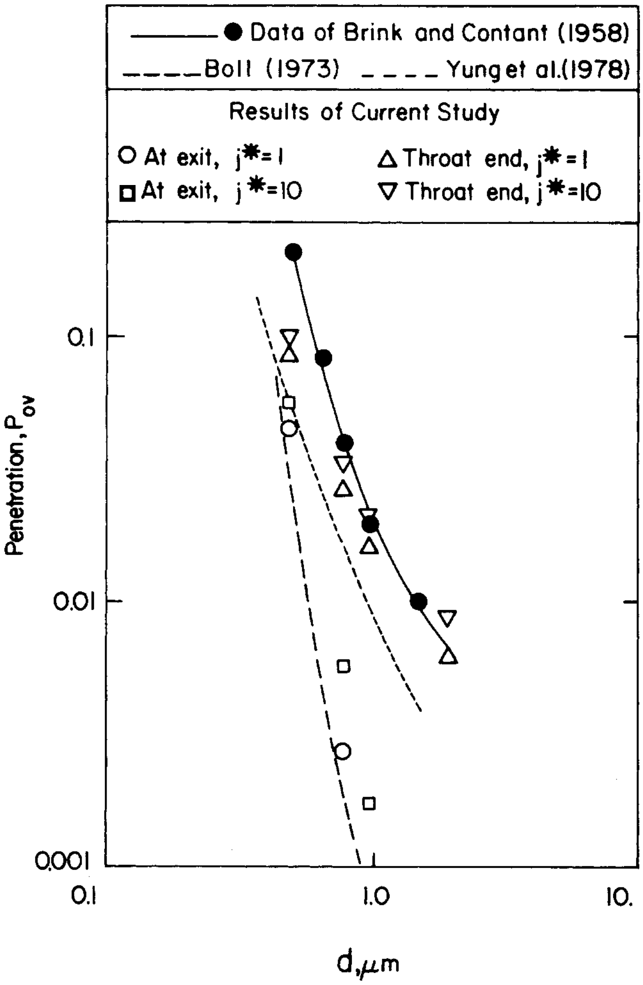

Figure 2. Comparison of several model predictions with experimental data obtained by Brink and Contant (1958).

change in the penetration.

Brink and Contant Scrubber. Brink and Contant (1958) obtained performance data on a Pease-Anthony venturi scrubber installed in a phosphoric acid plant. Their experimental design consisted of studying four main variables: throat gas velocity, spray liquid rate, spray liquid velocity, and number of injection jets. In one run, a cascade impactor was used to measure the particle size distribution at the venturi entrance and exit to establish the collection efficiency. The results are presented graphically in Figure 2 along with the predictions of several models.

In addition to the present study, two other models have been used to predict the performance of the scrubber used by Brink and Contant (1958). These are the throat model presented by Yung et al. (1978) and the variable geometry model presented by Boll (1973). It can be seen that the results predicted for the overall collection are higher than that observed by Brink and Contant (1958). In fact, the results obtained by Boll (1973) are essentially the same as that obtained assuming a drop size equal to the Sauter mean diameter. This over-estimation of predicted collection efficiency is probably related to the nonuniform distribution of drops across the venturi at the throat as mentioned by Boll (1973). The penetrations predicted by assuming a single drop class and 10 drop classes differed by 10-50%, indicating the need to account for the shape of the droplet size distribution.

Figure 2 also shows that the model of Yung et al. (1978) provided a better estimate of the penetration than did Boll's model or the model of this research. Since their model only considered particle collection to take place in the throat section, it is expected that it would produce lower efficiencies than an accurately represented geometry such as proposed in this research. This does not indicate that venturi scrubbers do not collect particles after the throat section as suggested by Yung et al. (1978), but without specifically modeling the poor distribution of spray liquid and other effects, a better estimate might be obtained in some cases by only considering collection to occur in the throat. If the penetrations at the end of the throat section predicted by the current model are examined (Figure 2), they also represent the Brink and Contant data very well.

Ekman and Johnstone Scrubber. Ekman and Johnstone (1951) made measurements of the collection efficiency of 1 μm diameter dibutyl-phthalate particles using a laboratory scale scrubber having a total length of approximately 0.5 m. In their experiments, water was introduced in three ways: a single jet injected downstream along the venturi axis; radially outward from a brass pipe with four 18.5 λ-mm holes; and radially inward from a single 32-mm jet at the throat entrance. Of these, it was felt that the latter configuration best represented the mode of operation of industrial scale venturi scrubbers.

Their results for the inward radial configuration are presented graphically in Figure 3, plotted as a function of the performance parameter $L\sqrt{St}$. This parameter has been traditionally selected as providing the most suitable variable for correlating performance data. In this paper, the Stokes number is defined in terms of the Sauter mean diameter and the gas velocity in the throat.

The predictions of the current model are also included in Figure 3. It can be seen that the model overestimates the collection efficiency (expressed as number of transfer units) by 50-100% in most cases. This is in keeping with previous studies (cf., Boll, 1973) where actual efficiencies were only 10-35% of the theoretical predictions. The reasons for this discrepancy are several. First, the scrubber used in the research was a laboratory scrubber, and scrubbers of this size are known to be susceptible to poor liquid distribution. Secondly, the spray was introduced by means of a single jet rather than multiple jets which

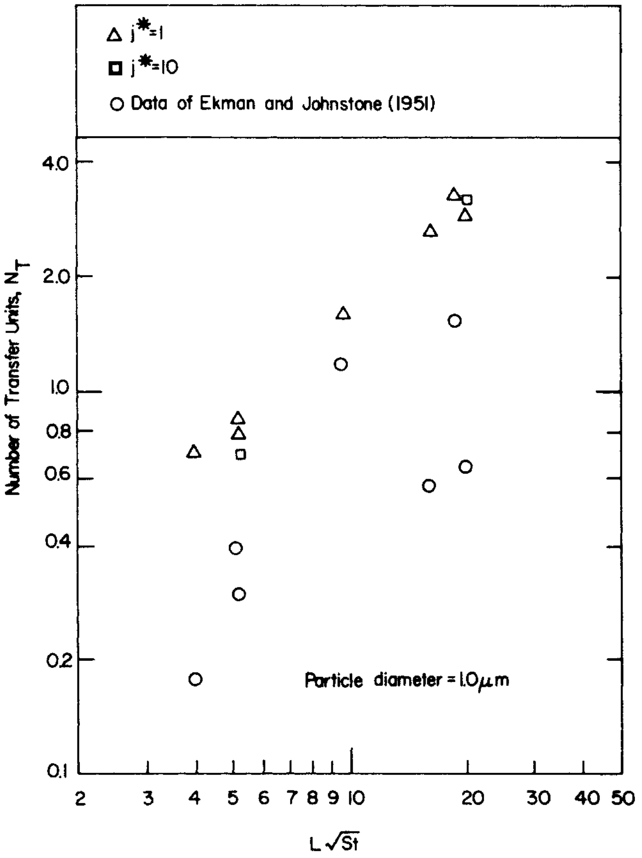

Figure 3. Comparison of model predictions with experimental data obtained by Ekman and Johnstone (1951).

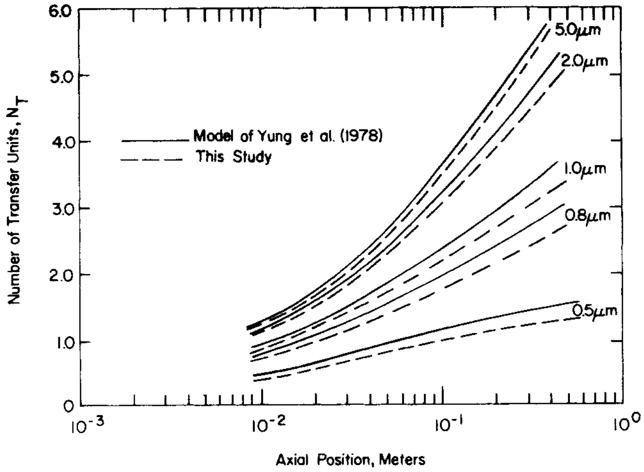

Figure 4. Comparison of present study with model results predicted by Yung et al. (1978).

could have aggravated this problem. Lastly, the scrubber was operated in a mode which produced values for $L\sqrt{St}$ which were less than 0.002, corresponding to very low liquid loading ratios. This corresponds to operating the venturi at collection efficiencies less than 60% in most cases, which is not the normal range of operation for industrial scrubbers. Taken together, these facts tend to limit the usefulness of this bench scale venturi to test models of industrial scale equipment. Unfortunately, data of the type and amount required for complete model validation do not exist in the open literature.

It can also be noted from Figure 3 that the data of Ekman and Johnstone cannot be very well correlated by the parameter $L\sqrt{St}$, while the predictions of the proposed model fall reasonably close to a straight line. For this particular geometry and range of operating conditions, the use of a single drop class or multiple drop classes did not seem to be as significant as in the case of the Brink and Contant data, perhaps due to the small values of the Sauter mean diameter at the low liquid loading ratios.

The Yung et al. Model. The present model was compared with the "state-of-the-art" analytical model of Yung et al. (1978). This particular model allows for a finite throat length rather than an infinite length as is assumed in other models. The major assumptions of this model are: 1) The venturi scrubber consists solely of a straight throat section with water injection at the entrance of the throat; 2) A single droplet size is used; 3) The initial velocity for the drop phase is zero; 4) No mass or heat transfer effects are considered; 5) Collection occurs by inertial impaction only, and a simplified empirical relationship is used to express the target efficiency in the potential flow regime ($E_{imp} = (K_p/K_p + 0.7)^2$ where the impaction parameter, $K_p = 2$ St); and 6) The drag coefficient is represented by a simplified empirical expression, $C_D = 18.5/Re^{0.6}$.

In order to provide a valid comparison, the following conditions were specified for the model proposed in this research: 1) The geometry was made consistent with that of Yung et al.; namely, the venturi consisted only of a straight throat section; 2) The water injection site was located at the entrance to the throat; and 3) A low mass and heat transfer situation was selected.

Figure 4 shows a comparison of the results obtained for each model for a liquid loading ratio of 0.0014 and a throat gas velocity of 60 m/s. It can be seen that the Yung et al. model overpredicts the collection efficiency at all points along the length of the venturi. However, in applying the model to actual scrubber geometries, the Yung et al. equation underpredicts the overall efficiency since it does not allow for collection to occur in the other areas of the venturi.

Effect of Operating Parameters on Performance

In order to demonstrate the effect of various parameters on the performance of the model, a reference or base case can be defined for comparison purposes. The operating conditions selected represent typical or average values from the range normally encountered in industrial applications. Except where otherwise indicated, the scrubber configuration simulated is that of Boll's prototype described previously.

The following conditions define the base case: $L = 0.0014$ m^3(spray)/m^3(gas); $v_{th} = 60$ m/s; $T_a = 350°$K; $T_d = 300°$K; $P_T = 101.38$ kPa; $H_0 = 0.108$ kg(water)/kg (dry air); $x_{inj} = 0.61$ m; and $v_{inj} = 5.0$ m/s. Under these conditions, the Sauter mean diameter is 175 μm, and the drop distribution can be represented by the following 10 diameter classes:

Class	Diameter Range	Mean Diameter	$f'(D_j)$
1	20-40 μm	30 μm	0.526
2	40-60 μm	50 μm	0.220
3	60-80 μm	70 μm	0.108
4	80-100 μm	90 μm	0.059
5	100-120 μm	110 μm	0.034
6	120-140 μm	130 μm	0.021
7	140-160 μm	150 μm	0.014
8	160-180 μm	170 μm	0.009
9	180-200 μm	190 μm	0.006
10	200-220 μm	210 μm	0.004

Two variables have long been recognized as most important in modeling scrubber performance. These are the liquid loading ratio and the gas velocity. Increasing the liquid loading ratio increases the Sauter mean diameter, thus tending to decrease the Stokes number (hence, target efficiency). In addition, it directly increases the concentration of droplets. The throat gas velocity affects the Sauter mean diameter also. A series of runs was made to examine the effect of both of these variables on scrubber performance assuming the drops to be a single size. The use of a single drop size is consistent with the other scrubber models that are available.

Effect of Gas Velocity. Figure 5 illustrates the effect of gas velocity, plotted as the number of transfer units vs. the parameter $L\sqrt{St}$. For a loading ratio of 0.008 and fixed injection site characteristics, increasing the throat velocity increased collection. The reasons for this are several. First, the Nukiyama-Tanasawa equation predicts that the spray will be broken into more and smaller droplets with increasing throat velocity. As both the higher velocity difference and the smaller droplet diameter tend to increase the Stokes number, this would correspond to higher impaction collection efficiency. Also, since more drops can be formed (from a given volume of liquid), their concentration is higher, which directly increases the collection rate. The figure also shows that this effect was most important for values of $L\sqrt{St}$ greater than 0.001. At values less than this, relatively little change occurred in the collection efficiency by increasing the velocity from 30 to 60 m/s.

For liquid loading ratios of 0.0014 and 0.0016, the same general conclusions apply to these conditions although the curves are somewhat steeper in slope at the higher liquid loading ratio.

Effect of Liquid Loading Ratio. Figure 6, a plot of the number of transfer units vs. $L\sqrt{St}$, shows that at a gas velocity of 30 m/s, increasing the liquid loading ratio can either increase or decrease the performance. At higher values of $L\sqrt{St}$, larger values of L effected improved performance since the larger volume of liquid produced a higher droplet concentration. However, since D_{32} varies as $L^{1.5}$ in the Nukiyama-Tanasawa equation, a trade-off exists since the larger drop size has a smaller Stokes number corresponding to a lower efficiency. The overall effect of increasing L at low values of the parameter $L\sqrt{St}$ was to decrease performance. For $v_{th} = 30$ m/s, the break point is approximately 0.002.

Similar behavior for throat gas velocities of 60 and 120 m/s

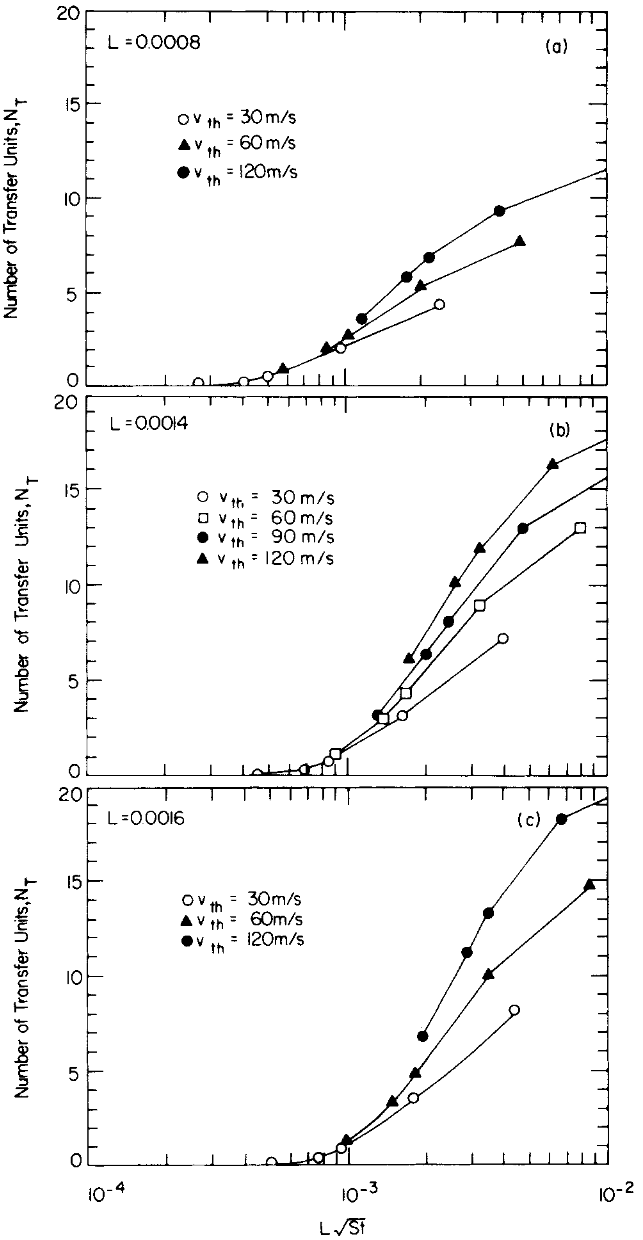

Figure 5. Effect of gas velocity on particle collection efficiency: (a) L = 0.0008, (b) L = 0.0014, (c) L = 0.0016.

was observed. In particular, at v_{th} = 120 m/s for the range of loading ratios and particle sizes examined, increasing the liquid loading ratio always produced an increase in collection. The break point is apparently at $L\sqrt{\mathrm{St}}$ less than 0.001.

Figure 7 summarizes all of the data presented in this section, and illustrates that at low values of $L\sqrt{\mathrm{St}}$ the effect of the parameters investigated was very slight. However, at values of $L\sqrt{\mathrm{St}}$ greater than 0.001 collection efficiency was quite sensitive to changes in gas flow rate and loading ratio.

Most of the remaining simulations discussed in this section were made using 10 droplet classes. This number was selected as providing a reasonable approximation to the continuous nature of the drop-size distribution while keeping the required computation time acceptable.

Effect of Drop Distribution Parameter δ. In discussing liquid atomization, it was noted that the parameter δ controls the shape of the droplet distribution function. Three runs were made to evaluate the effect of this parameter on the venturi collection efficiency using conditions described for the base case. The results are tabulated in Table 1 and show that there were only slight changes in performance as δ increased from 1/4 to 1/2. It appears, therefore, that the choice of δ is not critical for the case of venturi scrubbers as long as it is in the range specified. It may be possible, however, to produce sprays having the same Sauter mean diameter but with other values of δ (by other than pneumatic means) and produce improved results.

Effect of Transition Reynolds Number. As was discussed previously, the flow fields around the collector droplets encountered in scrubbing operations are not strictly potential or viscous in nature. Previous models have assumed the flow fields to be entirely potential. As suggested by the work of Tardos et al. (1978), intermediate behavior occurs in the range of 10 < Re < 40, and in the base case, a transition Reynolds number of 20 was selected to provide switching between the required target efficiency equations. Transition Reynolds numbers of 4 and 100 were also evaluated so as to include values which would reflect nearly total potential or viscous flow regimes.

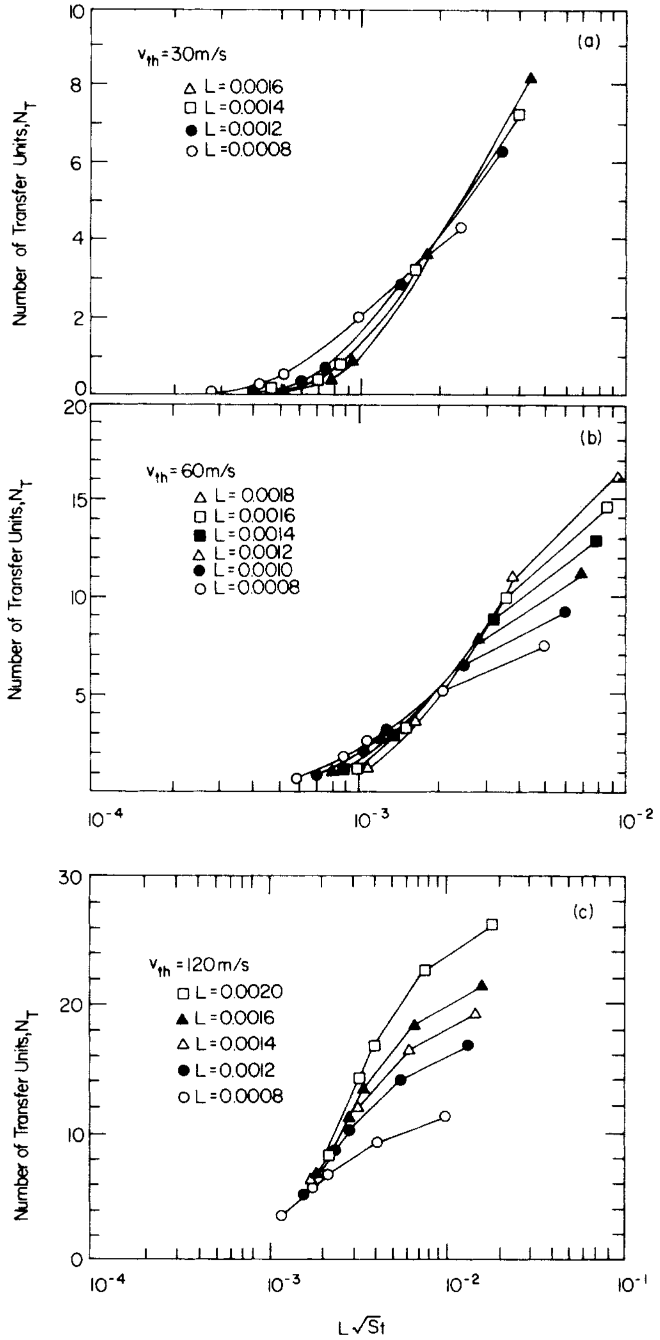

Figure 6. Effect of liquid loading ratio on the particle collection efficiency: (a) v_{th} = 30m/s, (b) v_{th} = 60m/s, (c) v_{th} = 120m/s.

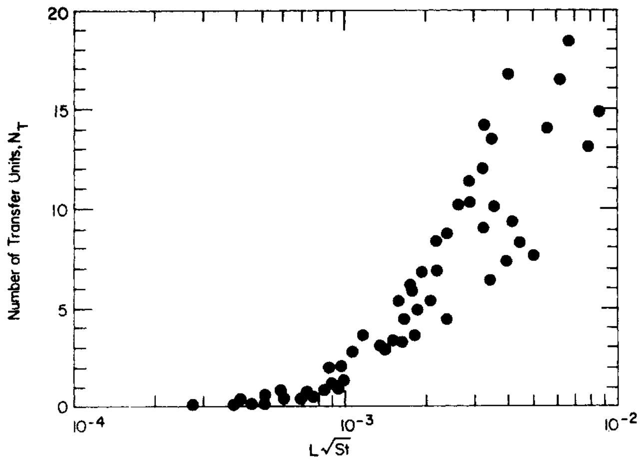

Figure 7. Summary of data for single drop simulations.

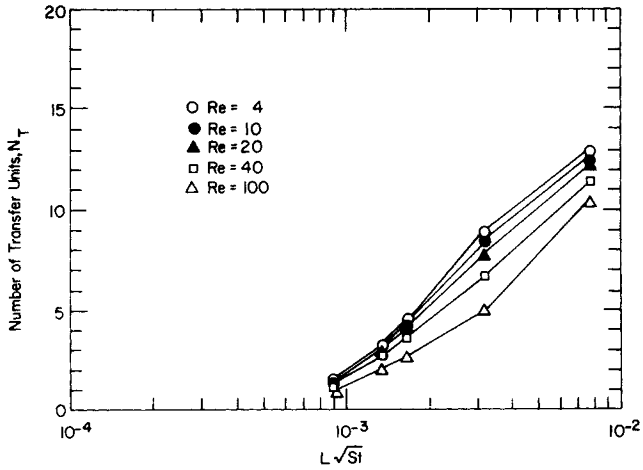

Figure 8. Effect of transition Reynolds number on particle collection efficiency.

TABLE 1. EFFECT OF δ ON THE COLLECTION EFFICIENCY OF PARTICLES OF VARIOUS DIAMETERS

	Number of Transfer Units				
δ	0.5 μm	0.8 μm	1.0 μm	2.0 μm	5.0 μm
1/4	1.379	3.052	4.143	7.843	12.25
1/3	1.345	3.071	4.205	8.026	12.37
1/2	1.326	3.111	4.292	8.269	12.53

The effects of this parameter are plotted in Figure 8. For transition Reynolds number between 40 and 100, the collection efficiency was moderately affected. This was especially true for the larger particles (corresponding to larger $L\sqrt{St}$). As the transition value decreased (corresponding to using the potential regime relationships for a greater portion of the time in the venturi), the efficiency increased as would be expected since the target efficiency for a given Stokes number is higher for potential flow. Since the present results show that the viscous flow regime can be encountered under normal operating conditions, this may explain why many models tend to overestimate performance.

Comparison Using Single and Multiple Drop Classes. A comparison was made using a distribution consisting of ten drop classes with that of a single droplet class. These results are shown in Figure 9. Under the conditions specified in the base case, using a single class slightly underestimated the collection efficiency at low values of the Stokes number, while it moderately overestimated the efficiency at higher Stokes numbers. This was probably due to changes in the target efficiency due both to impaction and interception considerations. It is conceivable that a single drop size could duplicate the multiple drop class results. However, it is difficult to determine *a priori* what that drop size should be.

Effect of Liquid Loading Ratio and Gas Velocity. The effect of liquid loading ratio and throat gas velocity was extensively evaluated in simulations involving a single drop class. A series of simulations was also made using ten drop classes to see if the same trends were predicted. Figure 10 shows the effect of liquid loading ratio at a throat velocity of 60 m/s and can be compared to Figure 6b. Although the same general conclusions applied to the multiple drop cases, the single drop model consistently overpredicted the collection efficiency especially at high values of $L\sqrt{St}$. Figure 11 shows the effect of throat gas velocity at a loading ratio of 0.0014 and can be compared to Figure 5b. In this case also, the general conclusions reached for the single drop cases were correct although the single drop model consistently estimated a higher collection efficiency.

Effect of Injection Parameters. The location of the injection site and the initial velocity of the liquid jet can affect performance. Figure 12 contains the results of simulations made to test the effect of injection site. For the particular geometry modeled, moving the location of the injection site varied the effective contact time of droplets in the venturi, and also altered the drop size distribution since the gas velocity used in the Nukiyama-Tanasawa equation will be a function of position in the scrubber. In this series of runs, the axial position of the

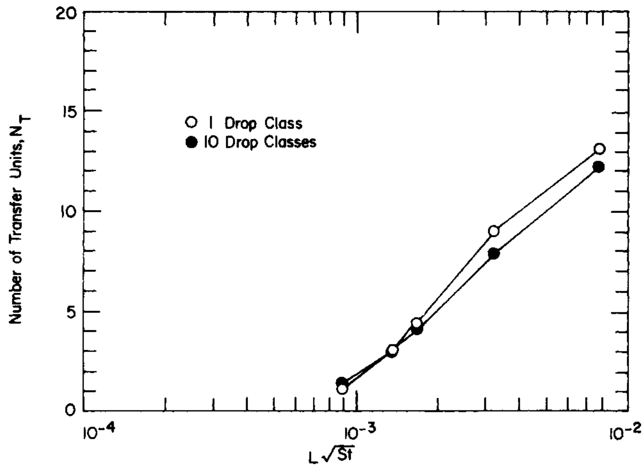

Figure 9. Effect of number of drop class intervals on particle collection efficiency.

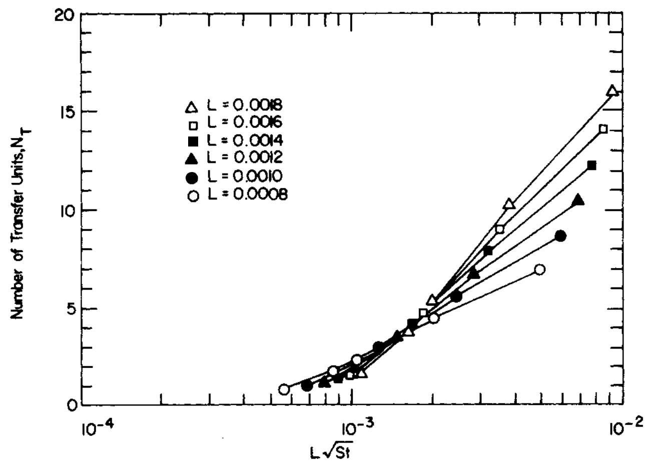

Figure 10. Effect of liquid loading ratio at v_{th} = 60m/s on particle collection efficiency using ten drop classes.

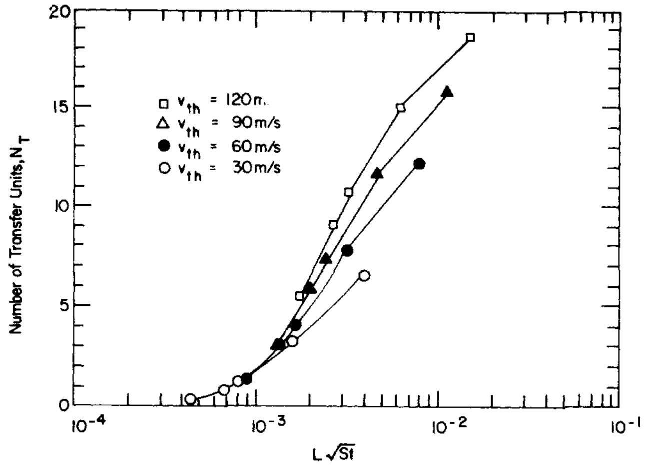

Figure 11. Effect of throat gas velocity at $L = 0.0014$ on particle collection efficiency using ten drop classes.

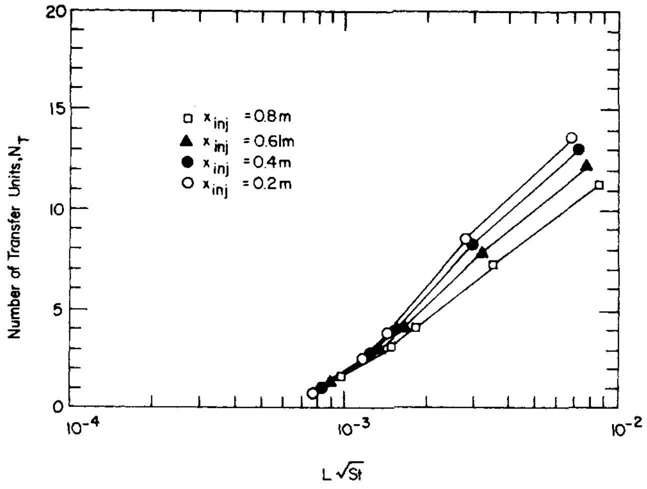

Figure 12. Effect of injection site location on particle collection efficiency using ten drop classes.

TABLE 2. EFFECT OF THE SAUTER MEAN DIAMETER ON THE COLLECTION EFFICIENCY OF PARTICLES OF VARIOUS DIAMETERS

Case	Number of Transfer Units				
	0.5 μm	0.8 μm	1.0 μm	2.0 μm	5.0 μm
N-T, $\overline{D_{32}}$ = 175 μm (j^* = 10)	1.379	3.052	4.143	7.843	12.25
N-T, $\overline{D_{32}}$ = 175 μm (j^* = 1)	1.200	3.080	4.396	8.976	13.11
Boll, $\overline{D_{32}}$ = 155 μm (j^* = 1)	1.328	3.265	4.609	9.167	13.24

injection site was moved from 0.2 m to 0.8 m from the beginning of the converging section. This provided a range of Sauter mean diameters of 231 to 148 μm. The effect of injection site location of performance was complex. For particles with diameters less than 1.0 μm, performance increased as the injection site was moved closer to the throat since the higher relative velocity at these positions allowed for better capture of these small particles. However, for the particles larger than 2.0 μm (those which are collected relatively easily), collection decreased as the injection site was moved toward the throat since the contact time was shorter.

The use of a nonzero droplet velocity at the injection site is required in one-dimensional modeling since the drop continuity equation predicts an infinite concentration for a zero drop velocity. To test the sensitivity of the model to using a small (and arbitrary) value for the initial drop velocity, runs were made at u_{inj} values of 2, 5, and 10 m/s. The results showed that only very slight differences existed among the values tested. It can be concluded that the use of any value less than 5 m/s is acceptable.

Effect of Throat Length. In optimizing the design of a scrubber, the length of the throat section should be very important. A longer throat affects performance since it allows more contact time between the liquid and dust. However, from an operating cost standpoint, too long a throat is undesirable since it consumes more mechanical energy in frictional losses to the walls.

Figure 13a summarizes results obtained using a throat length of 0.1, 0.3, and 1.0 m, and shows that increasing the throat length increased the collection for all sizes of particles. The effect was most pronounced for particles with larger Stokes numbers. In Figure 13b the same data are presented in terms of the parameter $N_{VH}\sqrt{St}$, where N_{VH} is the number of throat velocity heads defined as

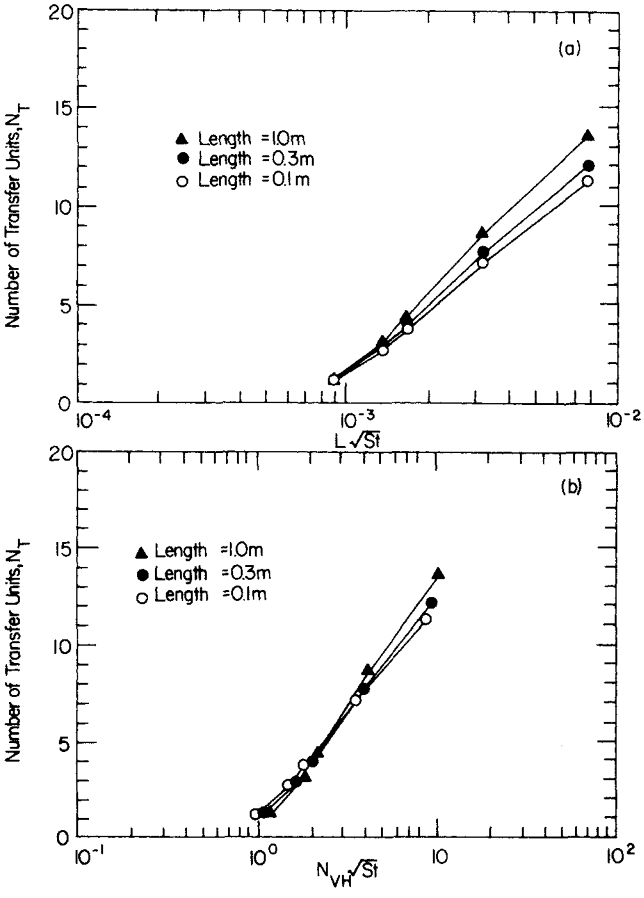

Figure 13. Effect of throat length on particle collection efficiency using ten drop classes: (a) shown as function of $L\sqrt{St}$; (b) shown as function $N_{VH}\sqrt{St}$.

$$N_{VH} = \frac{\Delta P_T}{\frac{1}{2}\rho v_{th}^2} \tag{22}$$

Although N_{VH} has been used in the literature, its usefulness is limited since the value of N_{VH} cannot generally be predicted without actually making a simulation. It can be observed that N_{VH} is proportional to the energy input to the venturi scrubber. For a given value of this performance parameter, operation can be either more or less efficient as the throat length is increased.

This is somewhat surprising since it has generally been assumed that increasing the energy input increases the collection efficiency.

Effect of Sauter Mean Diameter. As previously mentioned, some measurements indicate that the Sauter mean diameter predicted by the Nukiyama-Tanasawa equation may not represent the actual mean size present in the venturi (cf., Boll et al., 1974). Although the measurements which support this concept were taken at a point far removed from the injection site (and therefore, far from the major particle collection zone), the effect of this variable was studied. Table 2 contains the performance data for the base case assuming a Nukiyama-Tanasawa Sauter mean diameter and a Sauter mean diameter predicted by the Boll et al. relationship for a single drop class. The Boll et al. relationship resulted in higher collection efficiencies for the single drop class runs. This is consistent with the smaller $\overline{D_{32}}$ predicted (155 μm versus 175 μm) which would increase the Stokes number and the impaction target efficiency. Since Boll et al. did not discuss the shape of the drop distribution, only the single drop class was run. Further experimental work is required to determine if $\overline{D_{32}}$ is a function of the axial position.

NOTATION

A	= local venturi cross-sectional area, m^2
b	= parameter (Eq. 1), m$^{-\delta}$
C_D	= drag coefficient
C_f	= Cunningham slip correction factor
d	= particle diameter, m
D	= droplet diameter, m
D_h	= hydraulic diameter of venturi, m
$\overline{D_{32}}$	= Sauter mean diameter, m
E_{ij}	= local target efficiency for particles of the i^{th} class being collected by droplets of the j^{th} class
$E_{i_{ov}}$	= overall collection efficiency for particles of the i^{th} class
f	= friction factor
$f(D)$	= fraction of droplets with characteristic diameter D, m^{-1}
$f'(D_j)$	= fraction of droplets in size interval having characteristic diameter D_j
$\vec{F}_B$	= body force vector, N
H	= local humidity, kg (water vapor)/kg (dry air)
K_p	= impaction parameter, 2 St
L	= liquid to gas loading ratio, m^3 (liquid)/m^3 (gas)
$\dot{m}$	= rate of mass moving through the control volume, kg/s
M	= mass, kg
n	= particle concentration, particles/m^3
$\dot{n}$	= particle loading, particles/s
N	= droplet concentration, drops/m^3
$\dot{N}$	= droplet loading, drops/s
N_T	= number of transfer units
N_{VH}	= number of velocity heads
$P_{i_{ov}}$	= particle penetration for i^{th} class
P_T	= total pressure, Pa
Q	= flow rate, m^3/s
Re	= Reynolds number, $D\rho\lvert v - u\rvert/\mu$
St	= Stokes number, $C_f(\rho_p - \rho)d^2 v_{rel}/(18\mu\overline{D_{32}})$
t	= time, s
T	= temperature, °K
$\vec{u}$, u	= droplet velocity and magnitude of droplet velocity, m/s
$\vec{v}$, v	= gas velocity and magnitude of gas velocity, m/s
V_H	= humid volume, m^3 (wet gas)/kg (dry air)
V_{rel}	= relative velocity between liquid and gas phases at injection site, m/s
x	= distance measured along x-axis, m

Greek Letters

α	= void fraction
Γ	= gamma function
δ	= parameter (Eq. 1)
ρ	= density, kg/m^3

Subscripts

exit	= venturi exit conditions
g	= gas phase
i	= pertaining to the i^{th} particle class
i^*	= total number of particle classes
inj	= injection site
inlet	= venturi inlet conditions
j	= pertaining to the j^{th} droplet class
j^*	= total number of droplet classes
o	= initial conditions (at entrance)
ov	= overall
s	= spray phase
th	= throat conditions

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